Elliptic curves over number fields

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We define the elliptic curve $y^2 + xy + \phi y = x^3 + (\phi + 1)x^2 + \phi x$ over the field $\mathbb{Q}(\sqrt{5})$ where $\phi = \frac{1+\sqrt{5}}{2}$. We set $a = \sqrt{5}$, $nf = \mathbb{Q}(\sqrt{5})$. We want to check the BSD conjecture for E.

```
? nf  = nfinit(a^2-5);
? nf.disc
%2 = 5
? phi = (1+a)/2;
? E   = ellinit([1,phi+1,phi,phi,0],nf);
? N   = ellglobalred(E)[1]
%5 = [31,13;0,1]
? tor = elltors(E) \\ Z/8Z
%6 = [8,[8],[[-1,Mod(-1/2*a+1/2,a^2-5)]]]
? ellrootno(E)
%7 = 1
```

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```
? om = E.omega
%8 = [[3.05217315,-2.39884477*I],
%      [8.43805989,4.21902994-1.57216679*I]]
? per = om[1][1]*om[2][1];
? tam = elltamagawa(E)
%10 = 2
? bsd = (per*tam) / (tor[1]^2*sqrt(abs(nf.disc)))
%11 = 0.35992895949803944944002575466348575048
? ellbsd(E)
%12 = 0.35992895949803944944002575466348575048
? L1 = lfun(E,1)
%13 = 0.35992895949803944944002575466348575048
```

Elliptic curve over the rationals

```
? E = ellinit([0,0,1,-7,6]);
? ellidentify(E)
%15 = [["5077a1",[0,0,1,-7,6],[[-2,3],[-1,3],[0,2]]
? lfunorderzero(E)
%16 = 3
? ellratpoints(E,10)
%17 = [[-3,0],[-3,-1],[-2,3],[-2,-4],[-1,3],[-1,-4]
? G = ellgenerators(E)
%18 = [[-2,3],[-1,3],[0,2]]
```

Symmetric power of an elliptic curve

```
? R = matdet(ellheightmatrix(E,G))
%19 = 0.41714355875838396981711954461809339675
? ellbsd(E)*R
%20 = 1.7318499001193006897919750850601528450
? lfun(E,1,3)/3!
%21 = 1.7318499001193006897919750850601528450
```

Symmetric power of an elliptic curve

```
? L = lfunsympow(E,2); L[2..5]
%22 = [0,[0,0,1],3,257759929]
? z = lfun(L,2)
%23 = 7.546258095366608923770477395762974673 3
? om=E.omega[1]*imag(E.omega[2])
%24 = -3.073387226819025873544516424165260575 5
? z*2*Pi*5077/(om*(2*Pi)^2)
%25 = -1984.000000000000000000000000000000000
? ellmoddegree(E)
%26 = 1984
```

Genus-2 curve

For the genus-2 curve $y^2 + (x^3 + 1)y = x^2 + x$:

```
? L=lfungenus2([x^2+x,x^3+1]);
? L[2..5]
%43 = [0,[0,0,1,1],2,249]
? lfun(L,1)
%44 = 0.13154950701147875921340134301217526069
? lfunan(L,10)
%45 = [1,-2,-2,1,0,4,-1,0,4,0]
```