Pari-GP reference card

(PARI-GP version 2.16.2)

Note: optional arguments are surrounded by braces {}.
To start the calculator, type its name in the terminal: `gp`
To exit `gp`, type `quit`, `\q`, or `<C-D>` at prompt.

Help

describe function	$?function$
extended description	$??keyword$
list of relevant help topics	$???pattern$
name of GP-1.39 function f in GP-2.*	`whatnow`(f)

Input/Output

previous result, the result before	`%, %`, %``, etc.`
n-th result since startup	`%`n
separate multiple statements on line	`;`
extend statement on additional lines	`\`
extend statements on several lines	$\{seq_1;\ seq_2;\}$
comment	`/* ... */`
one-line comment, rest of line ignored	`\\ ...`

Metacommands & Defaults

set default d to *val*	`default`$(\{d\},\{val\})$
toggle timer on/off	`#`
print time for last result	`##`
print defaults	`\d`
set debug level to n	`\g` n
set memory debug level to n	`\gm` n
set n significant digits / bits	`\p` n, `\pb` n
set n terms in series	`\ps` n
quit GP	`\q`
print the list of PARI types	`\t`
print the list of user-defined functions	`\u`
read file into GP	`\r` *filename*
set debuglevel for domain D to n	`setdebug`(D, n)

Debugger / break loop

get out of break loop	`break` or `<C-D>`
go up/down n frames	`dbg_up`$(\{n\})$, `dbg_down`
set break point	`breakpoint()`
examine object o	`dbg_x`(o)
current error data	`dbg_err()`
number of objects on heap and their size	`getheap()`
total size of objects on PARI stack	`getstack()`

PARI Types & Input Formats

`t_INT`. Integers; hex, binary	$\pm 31;\ \pm\text{0x1F},\ \pm\text{0b101}$
`t_REAL`. Reals	± 3.14, 6.022 E23
`t_INTMOD`. Integers modulo m	`Mod`(n, m)
`t_FRAC`. Rational Numbers	n/m
`t_FFELT`. Elt in finite field $\mathbf{F}_q$	`ffgen`$(q, \text{'t})$
`t_COMPLEX`. Complex Numbers	$x + y * \text{I}$
`t_PADIC`. p-adic Numbers	$x + \text{O}(p\hat{\ }k)$
`t_QUAD`. Quadratic Numbers	$x + y * $ `quadgen`$(D, \{\text{'w}\})$
`t_POLMOD`. Polynomials modulo g	`Mod`(f, g)
`t_POL`. Polynomials	$a * x\hat{\ }n + \cdots + b$
`t_SER`. Power Series	$f + \text{O}(x\hat{\ }k)$
`t_RFRAC`. Rational Functions	f/g
`t_QFB`. Binary quadratic form	`Qfb`(a, b, c)
`t_VEC`/`t_COL`. Row/Column Vectors	$[x, y, z]$, $[x, y, z]$~
`t_VEC` integer range	`[1..10]`

`t_VECSMALL`. Vector of small ints	`Vecsmall`$([x, y, z])$
`t_MAT`. Matrices	$[a, b; c, d]$
`t_LIST`. Lists	`List`$([x, y, z])$
`t_STR`. Strings	`"abc"`
`t_INFINITY`. $\pm\infty$	`+oo, -oo`

Reserved Variable Names

$\pi \approx 3.14, \gamma \approx 0.57, C \approx 0.91, I = \sqrt{-1}$	`Pi, Euler, Catalan, I`
Landau's big-oh notation	`O`

Information about an Object, Precision

PARI type of object x	`type`(x)
length of x / size of x in memory	`#`x, `sizebyte`(x)
real precision / bit precision of x	`precision`(x), `bitprecision`(x)
p-adic, series prec. of x	`padicprec`(x, p), `serprec`(x, v)
current dynamic precision	`getlocalprec, getlocalbitprec`

Operators

basic operations	`+, -, *, /, ^, sqr`
$i \leftarrow i+1, i \leftarrow i-1, i \leftarrow i*j, \ldots$	`i++, i--, i*=j,...`
Euclidean quotient, remainder	$x\backslash/y,\ x\backslash y,\ x\%y$, `divrem`$(x, y)$
shift x left or right n bits	$x<<n,\ x>>n$ or `shift`$(x, \pm n)$
multiply by 2^n	`shiftmul`(x, n)
comparison operators	`<=, <, >=, >, ==, !=, ===, lex, cmp`
boolean operators (or, and, not)	`||, &&, !`
bit operations	`bitand, bitneg, bitor, bitxor, bitnegimply`
maximum/minimum of x and y	`max`(x, y), `min`(x, y)
sign of x (gives $-1, 0, 1$)	`sign`(x)
binary exponent of x	`exponent`(x)
derivative of f, 2nd derivative, etc.	$f', f'', \ldots$
differential operator	`diffop`$(f, v, d, \{n=1\})$
quote operator (formal variable)	`'x`
assignment	`x = `*value*
simultaneous assignment $x \leftarrow v[1], y \leftarrow v[2]$	`[x,y] = v`

Select Components

Caveat: components start at index $n = 1$.

n-th component of x	`component`(x, n)
n-th component of vector/list x	$x[n]$
components $a, a+1, \ldots, b$ of vector x	$x[a..b]$
(m, n)-th component of matrix x	$x[m, n]$
row m or column n of matrix x	$x[m,], x[, n]$
numerator/denominator of x	`numerator`(x), `denominator`(x)

Random Numbers

random integer/prime in $[0, N[$	`random`(N), `randomprime`(N)
get/set random seed	`getrand, setrand`(s)

Conversions

to vector, matrix, vec. of small ints	`Col/Vec,Mat,Vecsmall`
to list, set, map, string	`List, Set, Map, Str`
create $(x \bmod y)$	`Mod`(x, y)
make x a polynomial of v	`Pol`$(x, \{v\})$
variants of `Pol` *et al.*, in reverse order	`Polrev, Vecrev, Colrev`
make x a power series of v	`Ser`$(x, \{v\})$
convert x to simplest possible type	`simplify`(x)
object x with real precision n	`precision`(x, n)
object x with bit precision n	`bitprecision`(x, n)
set precision to p digits in dynamic scope	`localprec`(p)
set precision to p bits in dynamic scope	`localbitprec`(p)

Character strings

convert to TeX representation	`strtex`(x)
string from bytes / from format+args	`strchr, strprintf`
split string / join strings	`strsplit, strjoin`
convert time t ms. to h, m, s, ms format	`strtime`(t)

Conjugates and Lifts

conjugate of a number x	`conj`(x)
norm of x, product with conjugate	`norm`(x)
L^p norm of x (L^∞ if no p)	`normlp`$(x, \{p\})$
square of L^2 norm of x	`norml2`(x)
lift of x from Mods and p-adics	`lift, centerlift`(x)
recursive lift	`liftall`
lift all `t_INT` and `t_PADIC` ($\rightarrow$`t_INT`)	`liftint`
lift all `t_POLMOD` ($\rightarrow$`t_POL`)	`liftpol`

Lists, Sets & Maps

Sets (= row vector with strictly increasing entries w.r.t. `cmp`)

intersection of sets x and y	`setintersect`(x, y)
set of elements in x not belonging to y	`setminus`(x, y)
symmetric difference $x\Delta y$	`setdelta`(x, y)
union of sets x and y	`setunion`(x, y)
does y belong to the set x	`setsearch`$(x, y, \{flag\})$
set of all $f(x, y), x \in X, y \in Y$	`setbinop`(f, X, Y)
is x a set ?	`setisset`(x)

Lists. create empty list: $L = $ `List()`

append x to list L	`listput`$(L, x, \{i\})$
remove i-th component from list L	`listpop`$(L, \{i\})$
insert x in list L at position i	`listinsert`(L, x, i)
sort the list L in place	`listsort`$(L, \{flag\})$

Maps. create empty dictionary: $M = $ `Map()`

attach value v to key k	`mapput`(M, k, v)
recover value attach to key k or error	`mapget`(M, k)
is key k in the dict? (set v to $M(k)$)	`mapisdefined`$(M, k, \{\&v\})$
remove k from map domain	`mapdelete`(M, k)

GP Programming

User functions and closures

x, y are formal parameters; y defaults to `Pi` if parameter omitted;
z, t are local variables (lexical scope), z initialized to 1.

`fun(x, y=Pi) = my(z=1, t); `*seq*
`fun = (x, y=Pi) -> my(z=1, t); `*seq*

attach help message h to s	`addhelp`(s, h)
undefine symbol s (also kills help)	`kill`(s)

Control Statements (X: formal parameter in expression *seq*)

if $a \neq 0$, evaluate seq_1, else seq_2	`if`$(a, \{seq_1\}, \{seq_2\})$
eval. *seq* for $a \leq X \leq b$	`for`$(X = a, b, seq)$
...for $X \in v$	`foreach`(v, X, seq)
...for primes $a \leq X \leq b$	`forprime`$(X = a, b, seq)$
...for primes $\equiv a \pmod{q}$	`forprimestep`$(X = a, b, q, seq)$
...for composites $a \leq X \leq b$	`forcomposite`$(X = a, b, seq)$
...for $a \leq X \leq b$ stepping s	`forstep`$(X = a, b, s, seq)$
...for X dividing n	`fordiv`(n, X, seq)
...$X = [n, factor(n)], a \leq n \leq b$	`forfactored`$(X = a, b, seq)$
...as above, n squarefree	`forsquarefree`$(X = a, b, seq)$
...$X = [d, factor(d)], d \mid n$	`fordivfactored`(n, X, seq)
multivariable `for`, lex ordering	`forvec`$(X = v, seq)$

Pari-GP reference card

(PARI-GP version 2.16.2)

Column 1

loop over partitions of n	forpart($p = n, seq$)
...permutations of S	forperm(S, p, seq)
...subsets of $\{1, \ldots, n\}$	forsubset(n, p, seq)
...k-subsets of $\{1, \ldots, n\}$	forsubset($[n, k], p, seq$)
...vectors v, $q(v) \leq B$; $q > 0$	forqfvec(v, q, b, seq)
...$H < G$ finite abelian group	forsubgroup($H = G$)
evaluate seq until $a \neq 0$	until(a, seq)
while $a \neq 0$, evaluate seq	while(a, seq)
exit n innermost enclosing loops	break($\{n\}$)
start new iteration of n-th enclosing loop	next($\{n\}$)
return x from current subroutine	return($\{x\}$)

Exceptions, warnings

raise an exception / warning	error(), warning()
type of error message E	errname(E)
try seq_1, evaluate seq_2 on error	iferr(seq_1, E, seq_2)

Functions with closure arguments / results

number of arguments of f	arity(f)
select from v according to f	select(f, v)
apply f to all entries in v	apply(f, v)
evaluate $f(a_1, \ldots, a_n)$	call(f, a)
evaluate $f(\ldots f(f(a_1, a_2), a_3) \ldots, a_n)$	fold(f, a)
calling function as closure	self()

Sums & Products

sum $X = a$ to $X = b$, initialized at x	sum($X = a, b, expr, \{x\}$)
sum entries of vector v	vecsum(v)
product of all vector entries	vecprod(v)
sum $expr$ over divisors of n	sumdiv($n, X, expr$)
...assuming $expr$ multiplicative	sumdivmult($n, X, expr$)
product $a \leq X \leq b$, initialized at x	prod($X = a, b, expr, \{x\}$)
product over primes $a \leq X \leq b$	prodeuler($X = a, b, expr$)

Sorting

sort x by k-th component	vecsort($x, \{k\}, \{fl = 0\}$)
min. m of x ($m = x[i]$), max.	vecmin($x, \{\&i\}$), vecmax
does y belong to x, sorted wrt. f	vecsearch($x, y, \{f\}$)
$\prod g^x \to$ factorization ($\Rightarrow$ sorted, unique g)	matreduce(m)

Input/Output

print with/without \n, TEX format	print, print1, printtex
pretty print matrix	printp
print fields with separator	printsep($sep, \ldots$),, printsep1
formatted printing	printf()
write $args$ to file	write, write1, writetex($file, args$)
write x in binary format	writebin($file, x$)
read file into GP	read($\{file\}$)
...return as vector of lines	readvec($\{file\}$)
...return as vector of strings	readstr($\{file\}$)
read a string from keyboard	input()

Files and file descriptors

File descriptors allow efficient small consecutive reads or writes from or to a given file. The argument n below is always a descriptor, attached to a file in r(ead), w(rite) or a(ppend) mode.

get descriptor n for file $path$ in given $mode$	fileopen($path, mode$)
...from shell cmd output (pipe)	fileextern(cmd)
close descriptor	fileclose(n)
commit pending write operations	fileflush(n)
read logical line from file	fileread(n)
...raw line from file	filereadstr(n)
write s\n to file	filewrite(n, s)
...write s to file	filewrite1(n, s)

Column 2

Timers

CPU time in ms and reset timer	gettime()
CPU time in ms since gp startup	getabstime()
time in ms since UNIX Epoch	getwalltime()
timeout command after s seconds	alarm($s, expr$)

Interface with system

allocates a new stack of s bytes	allocatemem($\{s\}$)
alias old to new	alias(new, old)
install function from library	install($f, code, \{gpf\}, \{lib\}$)
execute system command a	system(a)
...and feed result to GP	extern(a)
...returning GP string	externstr(a)
get $VAR from environment	getenv("VAR")
expand env. variable in string	strexpand(x)

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use export for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate f on $x[1], \ldots, x[n]$	parapply(f, x)
evaluate closures $f[1], \ldots, f[n]$	pareval(f)
as select	parselect($f, A, \{flag\}$)
as sum	parsum($i = a, b, expr$)
as vector	parvector($n, i, \{expr\}$)
eval f for $i = a, \ldots, b$	parfor($i = a, \{b\}, f, \{r\}, \{f_2\}$)
...for each element x in v	parforeach($v, x, f, \{r\}, \{f_2\}$)
...for p prime in $[a, b]$	parforprime($p = a, \{b\}, f, \{r\}, \{f_2\}$)
...for $p = a$ mod q	parforprimestep($p = a, \{b\}, q, f, \{r\}, \{f_2\}$)
...for $i = a, a + s, \ldots, b$	parforstep($i = a, \{b\}, s, f, \{r\}, \{f_2\}$)
...multivariate	parforvec($X = v, f, \{r\}, \{f_2\}, \{flag\}$)
export x to parallel world	export(x)
...all dynamic variables	exportall()
frees exported value x	unexport(x)
...all exported values	unexportall()

Linear Algebra

dimensions of matrix x	matsize(x)
multiply two matrices	$x * y$
...assuming result is diagonal	matmultodiagonal(x, y)
concatenation of x and y	concat($x, \{y\}$)
extract components of x	vecextract($x, y, \{z\}$)
transpose of vector or matrix x	$x\sim$, mattranspose(x)
adjoint of the matrix x	matadjoint(x)
eigenvectors/values of matrix x	mateigen(x)
characteristic/minimal polynomial of x	charpoly(x), minpoly(x)
trace/determinant of matrix x	trace(x), matdet(x)
permanent of matrix x	matpermanent(x)
Frobenius form of x	matfrobenius(x)
QR decomposition	matqr(x)
apply matqr's transform to v	mathouseholder(Q, v)

Constructors & Special Matrices

$\{g(x): x \in v$ s.t. $f(x)\}$	[g(x) | x <- v, f(x)]
$\{x: x \in v$ s.t. $f(x)\}$	[x | x <- v, f(x)]
$\{g(x): x \in v\}$	[g(x) | x <- v]
row vec. of $expr$ eval'ed at $1 \leq i \leq n$	vector($n, \{i\}, \{expr\}$)
col. vec. of $expr$ eval'ed at $1 \leq i \leq n$	vectorv($n, \{i\}, \{expr\}$)
vector of small ints	vectorsmall($n, \{i\}, \{expr\}$)

Column 3

$[c, c \cdot x, \ldots, c \cdot x^n]$	powers($x, n, \{c = 1\}$)
$[1, 2^x, \ldots, n^x]$	dirpowers(n, x)
matrix $1 \leq i \leq m, 1 \leq j \leq n$	matrix($m, n, \{i\}, \{j\}, \{expr\}$)
define matrix by blocks	matconcat(B)
diagonal matrix with diagonal x	matdiagonal(x)
is x diagonal?	matisdiagonal(x)
$x \cdot$ matdiagonal(d)	matmuldiagonal(x, d)
$n \times n$ identity matrix	matid(n)
Hessenberg form of square matrix x	mathess(x)
$n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$	mathilbert(n)
$n \times n$ Pascal triangle	matpascal($n - 1$)
companion matrix to polynomial x	matcompanion(x)
Sylvester matrix of x and y	polsylvestermatrix(x, y)

Gaussian elimination

kernel of matrix x	matker($x, \{flag\}$)
intersection of column spaces of x and y	matintersect(x, y)
solve $MX = B$ (M invertible)	matsolve(M, B)
one sol of $M * X = B$	matinverseimage(M, B)
basis for image of matrix x	matimage(x)
columns of x *not* in matimage	matimagecompl(x)
supplement columns of x to get basis	matsupplement(x)
rows, cols to extract invertible matrix	matindexrank(x)
rank of the matrix x	matrank(x)
solve $MX = B$ mod D	matsolvemod(M, D, B)
image mod D	matimagemod(M, D)
kernel mod D	matkermod(M, D)
inverse mod D	matinvmod(M, D)
determinant mod D	matdetmod(M, D)

Lattices & Quadratic Forms

Quadratic forms

evaluate ${}^t x Q y$	qfeval($\{Q = id\}, x, y$)
evaluate ${}^t x Q x$	qfeval($\{Q = id\}, x$)
signature of quad form ${}^t y * x * y$	qfsign(x)
decomp into squares of ${}^t y * x * y$	qfgaussred(x)
eigenvalues/vectors for real symmetric x	qfjacobi(x)

HNF and SNF

upper triangular Hermite Normal Form	mathnf(x)
HNF of x where d is a multiple of det(x)	mathnfmod(x, d)
multiple of det(x)	matdetint(x)
HNF of $(x \mid$ diagonal(D))	mathnfmodid(x, D)
elementary divisors of x	matsnf(x)
q-rank from elementary divisors	snfrank(v, q)
elementary divisors of $\mathbf{Z}[a]/(f'(a))$	poldiscreduced(f)
integer kernel of x	matkerint(x)
$\mathbf{Z}$-module $\leftrightarrow$ $\mathbf{Q}$-vector space	matrixqz(x, p)

Lattices

LLL-algorithm applied to columns of x	qflll($x, \{flag\}$)
...for Gram matrix of lattice	qflllgram($x, \{flag\}$)
find up to m sols of qfnorm(x, y) $\leq b$	qfminim(x, b, m)
v, $v[i] :=$number of y s.t. qfnorm(x, y) $= i$	qfrep($x, B, \{flag\}$)
perfection rank of x	qfperfection(x)
find isomorphism between q and Q	qfisom(q, Q)
precompute for isomorphism test with q	qfisominit(q)
automorphism group of q	qfauto(q)

convert qfauto for GAP/Magma	qfautoexport$(G, \{flag\})$
orbits of V under $G \subset \text{GL}(V)$	qforbits(G, V)

Polynomials & Rational Functions

all defined polynomial variables	variables()
get var. of highest priority (higher than v)	varhigher$(name, \{v\})$
...of lowest priority (lower than v)	varlower$(name, \{v\})$

Coefficients, variables and basic operators

degree of f	poldegree(f)
coef. of degree n of f, leading coef.	polcoef(f,n), pollead
main variable / all variables in f	variable(f), variables(f)
replace x by y in f	subst(f,x,y)
evaluate f replacing vars by their value	eval(f)
replace polynomial expr. $T(x)$ by y in f	substpol(f,T,y)
replace $x_1,\ldots,x_n$ by $y_1,\ldots,y_n$ in f	substvec(f,x,y)
$f \in A[x]$; reciprocal polynomial $x^{\deg f} f\left(\frac{1}{x}\right)$	polrecip(f)
gcd of coefficients of f	content(f)
derivative of f w.r.t. x	deriv$(f, \{x\})$
...n-th derivative of f	derivn$(f,n,\{x\})$
formal integral of f w.r.t. x	intformal$(f, \{x\})$
formal sum of f w.r.t. x	sumformal$(f, \{x\})$

Constructors & Special Polynomials

interpolation polynomial at $(x[1], y[1]), \ldots, (x[n], y[n])$, evaluated at t, with error estimate e	polinterpolate$(x, \{y\}, \{t\}, \{\&e\})$
monic polynomial from roots r	polfromroots(r)
T_n/U_n, H_n	polchebyshev(n), polhermite(n)
P_n, $L_n^{(\alpha)}$	pollegendre(n), pollaguerre(n,a)
n-th cyclotomic polynomial Φ_n	polcyclo(n)
return n if $f = \Phi_n$, else 0	poliscyclo(f)
is f a product of cyclotomic polynomials?	poliscycloprod(f)
Zagier's polynomial of index (n,m)	polzagier(n,m)

Resultant, elimination

discriminant of polynomial f	poldisc(f)
find factors of poldisc(f)	poldiscfactors(f)
resultant $R = \text{Res}_v(f,g)$	polresultant$(f,g,\{v\})$
$[u,v,R]$, $xu + yv = \text{Res}_v(f,g)$	polresultantext$(x,y,\{v\})$
solve Thue equation $f(x,y) = a$	thue$(t,a,\{sol\})$
initialize t for Thue equation solver	thueinit(f)

Roots and Factorization (Complex/Real)

complex roots of f	polroots(f)
bound complex roots of f	polrootsbound(f)
number of real roots of f (in $[a,b]$)	polsturm$(f, \{[a,b]\})$
real roots of f (in $[a,b]$)	polrootsreal$(f, \{[a,b]\})$
complex embeddings of t_POLMOD z	conjvec(z)

Roots and Factorization (Finite fields)

factor f mod p, roots	factormod(f,p), polrootsmod
factor f over $\mathbf{F}_p[x]/(T)$, roots	factormod$(f,[T,p])$, polrootsmod
squarefree factorization of f in $\mathbf{F}_q[x]$	factormodSQF$(f, \{D\})$
distinct degree factorization of f in $\mathbf{F}_q[x]$	factormodDDF$(f, \{D\})$
factor n-th cyclotomic pol. Φ_n mod p	factormodcyclo(n,p)

Roots and Factorization (p-adic fields)

factor f over $\mathbf{Q}_p$, roots	factorpadic(f,p,r), polrootspadic
p-adic root of f congruent to a mod p	padicappr(f,a)
Newton polygon of f for prime p	newtonpoly(f,p)
Hensel lift $A/\text{lc}(A) = \prod_i B[i]$ mod p^e	polhensellift(A,B,p,e)
$T = \prod(x - z_i) \mapsto \prod(x - \omega(z_i)) \in \mathbf{Z}_p[x]$	polteichmuller(T,p,e)
extensions of $\mathbf{Q}_p$ of degree N	padicfields(p,N)

Roots and Factorization (Miscellaneous)

symmetric powers of roots of f up to n	polsym(f,n)
Graeffe transform of f, $g(x^2) = f(x)f(-x)$	polgraeffe(f)
factor f over coefficient field	factor(f)
cyclotomic factors of $f \in \mathbf{Q}[X]$	polcyclofactors(f)

Finite Fields

A finite field is encoded by any element (t_FFELT).

find irreducible $T \in \mathbf{F}_p[x]$, deg $T = n$	ffinit$(p,n,\{x\})$
Create t in $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$	t = ffgen$(T,'t)$
...indirectly, with implicit T	t = ffgen$(q,'t)$; T = t.mod
map m from $\mathbf{F}_q \ni a$ to $\mathbf{F}_{q^k} \ni b$	m = ffembed(a,b)
build $K = \mathbf{F}_q[x]/(P)$ extending $\mathbf{F}_q \ni a$,	ffextend(a,P)
evaluate map m on x	ffmap(m,x)
inverse map of m	ffinvmap(m)
compose maps $m \circ n$	ffcompomap(m,n)
x as polmod over codomain of map m	ffmaprel(m,x)
F^n over $\mathbf{F}_q \ni a$	fffrobenius(a,n)
#{monic irred. $T \in \mathbf{F}_q[x]$, deg $T = n$}	ffnbirred(q,n)

Formal & p-adic Series

truncate power series or p-adic number	truncate(x)
valuation of x at p	valuation(x,p)

Dirichlet and Power Series

Taylor expansion around 0 of f w.r.t. x	taylor(f,x)
Laurent series of closure F up to x^k	laurentseries(f,k)
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	serconvol(a,b)
$f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$	serlaplace(f)
reverse power series F so $F(f(x)) = x$	serreverse(f)
remove terms of degree $< n$ in f	serchop(f,n)
Dirichlet series multiplication / division	dirmul, dirdiv(x,y)
Dirichlet Euler product (b terms)	direuler$(p = a, b, expr)$

Transcendental and p-adic Functions

real, imaginary part of x	real(x), imag(x)
absolute value, argument of x	abs(x), arg(x)
square/nth root of x	sqrt(x), sqrtn$(x,n,\{\&z\})$
all n-th roots of 1	rootsof1(n)
FFT of $[f_0,\ldots,f_{n-1}]$	$w = $ fftinit(n), fft/fftinv(w,f)
trig functions	sin, cos, tan, cotan, sinc
inverse trig functions	asin, acos, atan
hyperbolic functions	sinh, cosh, tanh, cotanh
inverse hyperbolic functions	asinh, acosh, atanh
$\log(x)$, $\log(1+x)$, e^x, $e^x - 1$	log, log1p, exp, expm1
Euler Γ function, $\log \Gamma$, Γ'/Γ	gamma, lngamma, psi
half-integer gamma function $\Gamma(n + 1/2)$	gammah(n)
Riemann's zeta $\zeta(s) = \sum n^{-s}$	zeta(s)
$\sum_{1 \leq n \leq N} n^s$	dirpowerssum(N,s)
Hurwitz's $\zeta(s,x) = \sum (n+x)^{-s}$	zetahurwitz(s,x)
Lerch $\Phi(z,s,x) = \sum z^n (n+x)^{-s}$	lerchphi(z,s,x)
Lerch $L(s,x,t) = \Phi(e^{2i\pi t}, s, x)$	lerchzeta(s,x,t)
multiple zeta value (MZV), $\zeta(s_1,\ldots,s_k)$	zetamult$(s, \{T\})$
all MZVs for weight $\sum s_i = n$	zetamultall(n)
convert MZV id to $[s_1,\ldots,s_k]$	zetamultconvert$(f, \{flag\})$
MZV dual sequence	zetamultdual(s)
multiple polylog $Li_{s_1,\ldots s_k}(z_1,\ldots,z_k)$	polylogmult(s,z)

incomplete Γ function ($y = \Gamma(s)$)	incgam$(s,x,\{y\})$
complementary incomplete Γ	incgamc(s,x)
$\int_x^\infty e^{-t} dt/t$, $(2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$	eint1, erfc
elliptic integral of 1st and 2nd kind	ellK(k), ellE(k)
dilogarithm of x	dilog(x)
m-th polylogarithm of x	polylog$(m,x,\{flag\})$
U-confluent hypergeometric function	hyperu(a,b,u)
Hypergeometric $_pF_q(A,B;z)$	hypergeom(A,B,z)
Bessel $J_n(x)$, $J_{n+1/2}(x)$	besselj(n,x), besseljh(n,x)
Bessel I_ν, K_ν, H_ν^1, H_ν^2, Y_ν	(bessel)i, k, h1, h2, y
k-th zero of $J_\nu(x)$	besseljzero$(nu, \{k = 1\})$
k-th zero of $Y_\nu(x)$	besselyzero$(nu, \{k = 1\})$
Airy functions $A_i(x)$, $B_i(x)$	airy(x)
Lambert W: x s.t. $xe^x = y$	lambertw(y)
Teichmuller character of p-adic x	teichmuller(x)

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm\infty$ (decreasing at least as x^{-2}) or

$(x-a)^{-\alpha}$ singularity	$[a, \alpha]$
exponential decrease $e^{-\alpha|x|}$	$[\pm\infty, \alpha]$, $\alpha > 0$
slow decrease $|x|^\alpha$	...$\alpha < -1$
oscillating as $\cos(kx)$	$\alpha = k\text{I}$, $k > 0$
oscillating as $\sin(kx)$	$\alpha = -k\text{I}$, $k > 0$
numerical integration	intnum$(x = a, b, f, \{T\})$
weights T for intnum	intnuminit$(a, b, \{m\})$
weights T incl. kernel K	intfuncinit$(t = a, b, K, \{m\})$
integrate $(2i\pi)^{-1} f$ on circle $|z - a| = R$	intcirc$(x = a, R, f, \{T\})$

Other integration methods

n-point Gauss-Legendre	intnumgauss$(x = a, b, f, \{n\})$
weights for n-point Gauss-Legendre	intnumgaussinit$(\{n\})$
quasi-periodic function, period $2H$	intnumosc$(x = a, f, H)$
Romberg (low accuracy)	intnumromb$(x = a, b, f, \{flag\})$

Numerical summation

sum of series $f(n)$, $n \geq a$ (low accuracy)	suminf$(n = a, expr)$
sum of alternating/positive series	sumalt, sumpos
sum of series using Euler-Maclaurin	sumnum$(n = a, f, \{T\})$
...Sidi summation	sumnumsidi$(n = a, f)$
$\sum_{n \geq a} F(n)$, F rational function	sumnumrat(F, a)
...$\sum_{p \geq a} F(p^s)$	sumeulerrat$(F, \{s = 1\}, \{a = 2\})$
weights for sumnum, a as in DE	sumnuminit$(\{\infty, a\})$
sum of series by Monien summation	sumnummonien$(n = a, f, \{T\})$
weights for sumnummonien	sumnummonieninit$(\{\infty, a\})$
sum of series using Abel-Plana	sumnumap$(n = a, f, \{T\})$
weights for sumnumap, a as in DE	sumnumapinit$(\{\infty, a\})$
sum of series using Lagrange	sumnumlagrange$(n = a, f, \{T\})$
weights for sumnumlagrange	sumnumlagrangeinit

Products

product $a \leq X \leq b$, initialized at x	prod$(X = a, b, expr, \{x\})$
product over primes $a \leq X \leq b$	prodeuler$(X = a, b, expr)$
infinite product $a \leq X \leq \infty$	prodinf$(X = a, expr)$
$\prod_{n \geq a} F(n)$, F rational function	prodnumrat(F, a)
$\prod_{p \geq a} F(p^s)$	prodeulerrat$(F, \{s = 1\}, \{a = 2\})$

Other numerical methods

real root of f in $[a,b]$; bracketed root	$\text{solve}(X = a, b, f)$
...interval splitting, step s	$\text{solvestep}(X = a, b, s, f, \{flag = 0\})$
limit of $f(t)$, $t \to \infty$	$\text{limitnum(f, \{alpha\})}$
asymptotic expansion of f (rational)	$\text{asympnum(f, \{alpha\})}$
...$N + 1$ terms as floats	$\text{asympnumraw(f, N, \{alpha\})}$
numerical derivation w.r.t x: $f'(a)$	$\text{derivnum}(x = a, f)$
evaluate continued fraction F at t	$\text{contfraceval}(F, t, \{L\})$
power series to cont. fraction (L terms)	$\text{contfracinit}(S, \{L\})$
Padé approximant (deg. denom. $\leq B$)	$\text{bestapprPade}(S, \{B\})$

Elementary Arithmetic Functions

vector of binary digits of $\|x\|$	$\text{binary}(x)$
bit number n of integer x	$\text{bittest}(x, n)$
Hamming weight of integer x	$\text{hammingweight}(x)$
digits of integer x in base B	$\text{digits}(x, \{B = 10\})$
sum of digits of integer x in base B	$\text{sumdigits}(x, \{B = 10\})$
integer from digits	$\text{fromdigits}(v, \{B = 10\})$
ceiling/floor/fractional part	ceil, floor, frac
round x to nearest integer	$\text{round}(x, \{\&e\})$
truncate x	$\text{truncate}(x, \{\&e\})$
gcd/LCM of x and y	$\text{gcd}(x, y), \text{ lcm}(x, y)$
gcd of entries of a vector/matrix	$\text{content}(x)$

Primes and Factorization

extra prime table	$\text{addprimes}()$
add primes in v to prime table	$\text{addprimes}(v)$
remove primes from prime table	$\text{removeprimes}(v)$
Chebyshev $\pi(x)$, n-th prime p_n	$\text{primepi}(x), \text{ prime}(n)$
vector of first n primes	$\text{primes}(n)$
smallest prime $\geq x$	$\text{nextprime}(x)$
largest prime $\leq x$	$\text{precprime}(x)$
factorization of x	$\text{factor}(x, \{lim\})$
...selecting specific algorithms	$\text{factorint}(x, \{flag = 0\})$
$n = df^2$, d squarefree/fundamental	$\text{core}(n, \{fl\}), \text{ coredisc}$
certificate for (prime) N	$\text{primecert}(N)$
verifies a certificate c	$\text{primecertisvalid}(c)$
convert certificate to Magma/PRIMO	primecertexport
recover x from its factorization	$\text{factorback}(f, \{e\})$
$x \in \mathbf{Z}$, $\|x\| \leq X$, $\gcd(N, P(x)) \geq N$	$\text{zncoppersmith}(P, N, X, \{B\})$
divisors of N in residue class $r \bmod s$	$\text{divisorslenstra}(N, r, s)$

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$	$\text{omega}(n), \text{ bigomega}$
divisors of n / number of divisors $\tau(n)$	$\text{divisors}(n), \text{ numdiv}$
sum of (k-th powers of) divisors of n	$\text{sigma}(n, \{k\})$
Möbius μ-function	$\text{moebius}(x)$
Ramanujan's τ-function	$\text{ramanujantau}(x)$

Combinatorics

factorial of x	$x!$ or $\text{factorial}(x)$
binomial coefficient $\binom{x}{k}$	$\text{binomial}(x, \{k\})$
Bernoulli number B_n as real/rational	$\text{bernreal}(n), \text{ bernfrac}$
$[B_0, B_2, \ldots B_{2k}]$	$\text{bernvec}(k)$
Bernoulli polynomial $B_n(x)$	$\text{bernpol}(n, \{x\})$
Euler numbers	$\text{eulerfrac, eulerreal, eulervec}$
Euler polynomial $E_n(x)$	$\text{eulerpol}(n, \{x\})$
Eulerian polynomial $A_n(x)$	eulerianpol
Fibonacci number F_n	$\text{fibonacci}(n)$
Harmonic number $H_{n,r} = 1^{-r} + \ldots + n^{-r}$	$\text{harmonic}(n, r)$
Stirling numbers $s(n, k)$ and $S(n, k)$	$\text{stirling}(n, k, \{flag\})$

number of partitions of n	$\text{numbpart}(n)$
k-th permutation on n letters	$\text{numtoperm}(n, k)$
...index k of permutation v	$\text{permtonum}(v)$
order of permutation p	$\text{permorder}(p)$
signature of permutation p	$\text{permsign}(p)$
cyclic decomposition of permutation p	$\text{permcycles}(p)$

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, $\mathbf{F}_q^*$

Euler ϕ-function	$\text{eulerphi}(x)$
multiplicative order of x (divides o)	$\text{znorder}(x, \{o\}), \text{ fforder}$
primitive root mod q / $x.\text{mod}$	$\text{znprimroot}(q), \text{ ffprimroot}(x)$
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	$\text{znstar}(x)$
discrete logarithm of x in base g	$\text{znlog}(x, g, \{o\}), \text{ fflog}$
Kronecker-Legendre symbol $(\frac{x}{y})$	$\text{kronecker}(x, y)$
quadratic Hilbert symbol (at p)	$\text{hilbert}(x, y, \{p\})$

Euclidean algorithm, continued fractions

CRT: solve $z \equiv x$ and $z \equiv y$	$\text{chinese}(x, y)$
minimal u, v so $xu + yv = \gcd(x, y)$	$\text{gcdext}(x, y)$
half-gcd algorithm	$\text{halfgcd}(x, y)$
continued fraction of x	$\text{contfrac}(x, \{b\}, \{lmax\})$
last convergent of continued fraction x	$\text{contfracpnqn}(x)$
rational approximation to x (den. $\leq B$)	$\text{bestappr}(x, \{B\})$
recognize $x \in \mathbf{C}$ as polmod mod $T \in \mathbf{Z}[X]$	$\text{bestapprnf}(x, T)$

Miscellaneous

integer square / n-th root of x	$\text{sqrtint}(x), \text{ sqrtnint}(x, n)$
largest integer e s.t. $b^e \leq b$, $e = \lfloor \log_b(x) \rfloor$	$\text{logint}(x, b, \{\&z\})$

Characters

Let $cyc = [d_1, \ldots, d_k]$ represent an abelian group $G = \oplus(\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure G affording a .cyc method; e.g. $\text{znstar}(q, 1)$ for Dirichlet characters. A character χ is coded by $[c_1, \ldots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$.

$\chi \cdot \psi$; χ^{-1}; $\chi \cdot \psi^{-1}$; χ^k	$\text{charmul, charconj, chardiv, charpow}$
order of χ	$\text{charorder}(cyc, \chi)$
kernel of χ	$\text{charker}(cyc, \chi)$
$\chi(x)$, G a GP group structure	$\text{chareval}(G, \chi, x, \{z\})$
Galois orbits of characters	$\text{chargalois}(G)$

Dirichlet Characters

initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$	$\text{G = znstar}(q, 1)$
convert datum D to $[G, \chi]$	$\text{znchar}(D)$
is χ odd?	$\text{zncharisodd}(G, \chi)$
real $\chi \to$ Kronecker symbol $(D/.)$	$\text{znchartokronecker}(G, \chi)$
conductor of χ	$\text{zncharconductor}(G, \chi)$
$[G_0, \chi_0]$ primitive attached to χ	$\text{znchartoprimitive}(G, \chi)$
induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$	$\text{zncharinduce}(G, \chi, N)$
χp	$\text{znchardecompose}(G, \chi, p)$
$\prod_{p\|(Q,N)} \chi p$	$\text{znchardecompose}(G, \chi, Q)$
complex Gauss sum $G_a(\chi)$	$\text{znchargauss}(G, \chi)$

Conrey labelling

Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \to$ character	$\text{znconreychar}(G, m)$
character $\to$ Conrey label	$\text{znconreyexp}(G, \chi)$
log on Conrey generators	$\text{znconreylog}(G, m)$
conductor of χ (χ_0 primitive)	$\text{znconreyconductor}(G, \chi, \{\chi_0\})$

True-False Tests

is x the disc. of a quadratic field?	$\text{isfundamental}(x)$
is x a prime?	$\text{isprime}(x)$
is x a strong pseudo-prime?	$\text{ispseudoprime}(x)$
is x square-free?	$\text{issquarefree}(x)$
is x a square?	$\text{issquare}(x, \{\&n\})$
is x a perfect power?	$\text{ispower}(x, \{k\}, \{\&n\})$
is x a perfect power of a prime? ($x = p^n$)	$\text{isprimepower}(x, \&n)$
...of a pseudoprime?	$\text{ispseudoprimepower}(x, \&n)$
is x powerful?	$\text{ispowerful}(x)$
is x a totient? ($x = \varphi(n)$)	$\text{istotient}(x, \{\&n\})$
is x a polygonal number? ($x = P(s, n)$)	$\text{ispolygonal}(x, s, \{\&n\})$
is pol irreducible?	$\text{polisirreducible}(pol)$

Graphic Functions

crude graph of $expr$ between a and b	$\text{plot}(X = a, b, expr)$

High-resolution plot (immediate plot)

plot $expr$ between a and b	$\text{ploth}(X = a, b, expr, \{flag\}, \{n\})$
plot points given by lists lx, ly	$\text{plothraw}(lx, ly, \{flag\})$
terminal dimensions	$\text{plothsizes}()$

Rectwindow functions

init window w, with size x,y	$\text{plotinit}(w, x, y)$
erase window w	$\text{plotkill}(w)$
copy w to w_2 with offset (dx, dy)	$\text{plotcopy}(w, w_2, dx, dy)$
clips contents of w	$\text{plotclip}(w)$
scale coordinates in w	$\text{plotscale}(w, x_1, x_2, y_1, y_2)$
ploth in w	$\text{plotrecth}(w, X = a, b, expr, \{flag\}, \{n\})$
plothraw in w	$\text{plotrecthraw}(w, data, \{flag\})$
draw window w_1 at (x_1, y_1), ...	$\text{plotdraw}([[w_1, x_1, y_1], \ldots])$

Low-level Rectwindow Functions

set current drawing color in w to c	$\text{plotcolor}(w, c)$
current position of cursor in w	$\text{plotcursor}(w)$
write s at cursor's position	$\text{plotstring}(w, s)$
move cursor to (x, y)	$\text{plotmove}(w, x, y)$
move cursor to $(x + dx, y + dy)$	$\text{plotrmove}(w, dx, dy)$
draw a box to (x_2, y_2)	$\text{plotbox}(w, x_2, y_2)$
draw a box to $(x + dx, y + dy)$	$\text{plotrbox}(w, dx, dy)$
draw polygon	$\text{plotlines}(w, lx, ly, \{flag\})$
draw points	$\text{plotpoints}(w, lx, ly)$
draw ellipse	$\text{plotarc}(w, lx, ly, \{flag\})$
draw line to $(x + dx, y + dy)$	$\text{plotrline}(w, dx, dy)$
draw point $(x + dx, y + dy)$	$\text{plotrpoint}(w, dx, dy)$

Convert to Postscript or Scalable Vector Graphics

The format f is either "ps" or "svg".

as ploth	$\text{plothexport}(f, X = a, b, expr, \{flag\}, \{n\})$
as plothraw	$\text{plothrawexport}(f, lx, ly, \{flag\})$
as plotdraw	$\text{plotexport}(f, [[w_1, x_1, y_1], \ldots])$